# Pairwise Comparison Models 

A Two-Tiered Approach for Predicting Wins and Losses in the NBA

## Motivation

- Test Bradley Terry model as the basis for finding strong predictive models for NBA games
- Test the success of an indirect, two-tiered approach to predicting wins
- Only using Win/Loss record might not be optimal


## Hypothesis

- Could be more effective to use a twotiered approach
- First identify the broad features that have high correlation with win rate
- Model wins based off of those features
- Predict for those features first and then predict wins


## Dean Oliver's Four Factors

- Effective Field Goal Percentage = (Field Goals Made $+0.5^{*}$ Three Pointers Made)/Field Goals Attempted
- Turnover Percentage =

Turnovers/(Field Goals Attempted + 0.44*Free Throw Attempts + Turnovers)

- Offensive Rebound Rate =

Offensive Rebounds/(Offensive Rebounds + Opposition Defensive Rebounds)

- Defensive Rebound Rate =

Defensive Rebound Rate = Defensive Rebounds/(Opposition Offensive Rebounds + Defensive Rebounds)

- Free Throw Factor =

Free Throws Made/Field Goals Attempted

## Bradley Terry Application

- The four factors are all rates
- To calculate A's turnover rate against B, we need

1. A's mean turnover rate
2. The league's mean turnover rate
3. The mean turnover rate of teams that play against B

## Why Bradley Terry?

- Simple
- Very little data required (only at the team level)
- Far fewer features to predict


## Methodology

- Data set 2010-11 NBA season
- $(82 * 30) / 2=1230$ observations
- 861 in training set and 369 in test set (70\%/30\%)


## Models

- Two predictive layers in model
- a model for predicting the four factors
- a model for predicting win rate from the four factors
- Reference model
- Only uses win/loss record to predict win rate


## Predicting Four Factors

- Only predict on a game using past games
- How many games to include in training sample?
- Two possible options
- Use every game leading up to prediction game
- Use a moving window of size $d$ games to predict


## Tuning window size

- Objective: tune $d$ with training set
- Set d=1, 2, 5, 10, 20
- Train on different number of observations
- E.g. when $\mathrm{d}=1$, I start training when every team has played at least 1 game
- Compute MSE for the 5 values of $d$ and also for the case in which every game is included


## Results

| Window <br> Size | num <br> obs. | Rebound <br> MSE | Turnover <br> MSE | eFG\% MSE | FT factor <br> MSE | Sum of MSE |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 844 | 0.016501403 | 0.002960085 | 0.011684333 | 0.022131734 | 0.053277555 |
| 2 | 776 | 0.011073513 | 0.002020287 | 0.007479058 | 0.02408846 | 0.044661318 |
| 5 | 693 | 0.007100297 | 0.00142125 | 0.005043673 | 0.01293233 | 0.02649755 |
| $\mathbf{1 0}$ | $\mathbf{5 3 6}$ | $\mathbf{0 . 0 0 6 3 6 2 8}$ | $\mathbf{0 . 0 0 1 2 4 9 4 1 9}$ | $\mathbf{0 . 0 0 4 4 3 2 8 8 3}$ | $\mathbf{0 . 0 0 2 7 7 6 6 6 5}$ | $\mathbf{0 . 0 1 4 8 2 1 7 6 7}$ |
| 20 | 371 | 0.005733524 | 0.001195112 | 0.004259816 | 0.005780949 | 0.016969401 |
| All games | 844 | 0.00608761 | 0.001254227 | 0.004407891 | 0.009369296 | 0.021119024 |

## Predicting wins from four factors

- Linear models
- Least squares regression
- Logistic regression
- Non-linear models
- Regression tree
- Classification tree
- Point differential vs Win/Loss
- Multicollinearity with Rebound features


## Feature Set



## Model Selection

- 10 -fold cross validation, i.e. randomly divide training set into 10 folds



## Results

- Best model is logistic regression with a moving window of 10 games

10-Fold Cross Validation

| Model | MSE | abs_y_hat -y$)$ | $0-1$ Loss |
| :--- | :--- | :--- | :--- |
| Least squares | 9.84896582 | 2.54035922 | 0.04298316 |
| Logistic regression | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | 0.03716921 |
| Regression tree | 74.90737 | 6.877177 | 0.2078856 |
| Classification tree | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | 0.1962978 |

## Logistic Regression Model

## Coefficients:

| (Intercept) | -28.976 | 4.716 | -6.145 | $8.01 \mathrm{e}-10$ | *** |
| :--- | ---: | ---: | ---: | ---: | ---: |
| TurnoverRate | -109.504 | 13.350 | -8.203 | $2.35 \mathrm{e}-16$ | *** |
| EFGRate | 111.361 | 12.237 | 9.101 | $<2 \mathrm{e}-16$ | *** |
| FreeThrowRate | 26.971 | 3.572 | 7.550 | $4.35 \mathrm{e}-14$ | *** |
| OffReboundRate | 30.590 | 4.257 | 7.186 | $6.69 \mathrm{e}-13$ | *** |
| DefReboundRate | 30.117 | 4.300 | 7.005 | $2.48 \mathrm{e}-12$ | *** |
| OppTurnoverRate | 97.897 | 11.676 | 8.384 | $<2 \mathrm{e}-16$ | *** |
| OppEFGRate | -109.903 | 11.910 | -9.228 | $<2 \mathrm{e}-16$ | *** |
| OppFreeThrowRate | -27.710 | 3.875 | -7.151 | $8.64 \mathrm{e}-13$ | *** |

## Tune single-tier model

- Compare 0-1 Loss
- Select window size of 20 games

| Window <br> Size | num <br> obs. | $0-1$ Loss |
| :--- | :--- | :--- |
| 1 | 844 | 0.4490521 |
| 2 | 776 | 0.4379562 |
| 5 | 693 | 0.4007732 |
| 10 | 536 | 0.3708514 |
| $\mathbf{2 0}$ | $\mathbf{3 7 1}$ | $\mathbf{0 . 3 4 5 1 4 9 3}$ |
| All games | 844 | 0.3414948 |

## Performance on test set

- Test set of 369 observations

| Model | 0-1 Loss | Correct Guesses | Total Games |
| :--- | :--- | :--- | :--- |
| Two-tier model | 0.3604336 | 236 | 369 |
| Single-tier win/loss | 0.3848238 | 227 | 369 |

## Compare with other popular models

- Omidiran
- 0-1 Loss
- Dummy model
- Home court advantage 0.4024
- Plus-minus models
- Least squares 0.4073
- Ridge regression 0.3732


## Compare with other popular models

- Errors seem to be at least as small as errors in the plus-minus model
- However, motivation of APM is to measure player performance
- But, our models require far fewer features


## Conclusion

- Reasonable evidence that models that indirectly predict wins can be successful
- Bradley Terry model can be applied beyond win/loss record
- Sample size in predicting game, i.e. window size

